

**Erratum: Implicit, partially linearized, electromagnetic particle simulation  
of plasma drift-wave turbulence  
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A linear electromagnetic “flutter” term was omitted in Eqs. (2a) and (6) involving the inner product of the parallel velocity along the electromagnetically perturbed field line with the gradient of the equilibrium Maxwellian distribution function. However, the particle trajectory equations implemented in the numerical simulations did not have this error; only the published equations had this omission. The direct-implicit particle simulation results reported were correct, and the implicit-moment equation simulations had a small  $O(\beta\Delta t_i)$  inaccuracy, where  $\beta$  is the ratio of the plasma pressure to the magnetic energy density and  $\Delta t_i$  is the time step. The corrected version of Eq. (2a) in Sec. II is

$$\frac{\partial \delta f}{\partial t} + v_{\parallel} \hat{\mathbf{b}} \cdot \frac{\partial \delta f}{\partial \mathbf{R}} - \frac{c}{B} \frac{\partial}{\partial \mathbf{R}} \cdot \left[ \left( \frac{\partial \bar{\phi}}{\partial \mathbf{R}} \times \hat{\mathbf{b}} \right) \delta f \right] = \frac{qv_{\parallel}}{T} \bar{\mathbf{E}} \cdot \hat{\mathbf{b}} F_M - \kappa \left( \frac{c}{B} \frac{\partial \bar{\phi}}{\partial y} - \frac{v_{\parallel}}{B} \frac{\partial A_z}{\partial y} \right) F_M, \quad (2a)$$

where  $\kappa \equiv -\nabla \ln F_M$ ,  $F_M$  is an equilibrium Maxwellian distribution function, and  $\hat{\mathbf{b}} = \hat{\mathbf{z}} + \hat{\mathbf{y}} B_y^{(0)}/B + \nabla \times A_z \hat{\mathbf{z}}/B$ . The second term inside the parentheses on the right side of Eq. (2a) is the linear flutter term.

The evolution equations for the particle weights of the electrons and ions given after Eq. (5) in Sec. II A inherited the error in the published Eq. (2a). The corrected versions of these equations are given as follows. Again, the direct-implicit and implicit-moment algorithms used in the numerical simulations, whose results were shown in the publication, used the corrected equations that follow. The ion weights  $w_j$  evolve according to

$$\dot{w}_j = \frac{qv_{\parallel}}{T} \bar{\mathbf{E}} \cdot \hat{\mathbf{b}} - \kappa_i \left( \frac{c}{B} \frac{\partial \bar{\phi}}{\partial y} - v_{\parallel} \frac{\delta B_x}{B} \right)$$

along the characteristics for the ions, where  $\delta B_x = \partial A_z / \partial y$ , and

$$\dot{w}_j = \frac{\dot{p}_{\parallel e} v_{\parallel}}{v_e^2} - \kappa_e \left( \frac{c}{B} \frac{\partial \bar{\phi}}{\partial y} - v_{\parallel} \frac{\delta B_x}{B} \right)$$

for the electrons.

The implicit-moment equations in Sec. II B were computed from the partially linearized drift-kinetic Vlasov equation, Eq. (6), that omitted the same linear flutter term as did the published Eq. (2a). The corrected electron drift-kinetic equation integrated with respect to  $v_{\perp}$  is

$$\frac{\partial \delta f_e}{\partial t} + v_{\parallel} \hat{\mathbf{b}} \cdot \nabla \delta f_e + \mathbf{v}_{\mathbf{E} \times \mathbf{B}} \cdot \nabla \delta f_e + \frac{ev_{\parallel}}{m_e v_e^2} F_{Me} \hat{\mathbf{b}} \cdot \mathbf{E} + \left( \frac{c}{B} \frac{\partial \bar{\phi}}{\partial y} - v_{\parallel} \frac{\delta B_x}{B} \right) \frac{F_{Me}}{L_{Te}} \left( \frac{L_{Te}}{L_{ne}} + \frac{v_{\parallel}^2}{2v_e^2} - \frac{1}{2} \right) = 0. \quad (6)$$

The moment equation for the perturbed electron density in the publication, Eq. (7), is unchanged in the presence of the flutter term. However, the moment equation for the parallel current per unit electron charge, Eq. (8), picks up an additional term (the second term on the right-hand side of the equality):

$$\begin{aligned} & \frac{j_{e\parallel}^{n+1} - j_{e\parallel}^n}{\Delta t_i} + v_e^2 \nabla_{\parallel} [\epsilon_2 n_e^{n+1} + (1 - \epsilon_2) n_e^n] + \nu_{ei} [\epsilon_2 j_{e\parallel}^{n+1} + (1 - \epsilon_2) j_{e\parallel}^n] - \frac{en_0}{m_e} \nabla_{\parallel} [\epsilon_2 \phi^{n+1} + (1 - \epsilon_2) \phi^n] - \frac{en_0}{m_e c} \frac{A_z^{n+1} - A_z^n}{\Delta t_i} \\ & = \left[ -\mathbf{v}_{\mathbf{E} \times \mathbf{B}} \cdot \nabla_{\perp} j_{e\parallel} + \frac{\delta B_x}{B} \frac{v_e^2}{L_{Te}} \left( \frac{L_{Te}}{L_{ne}} + 1 \right) + \nu_{ei} \bar{j}_{i\parallel} - \nabla_{\parallel} (P_{e\parallel}^{n+1/2} - n_e v_e^2) \right]^{**}. \end{aligned} \quad (8)$$

The flutter term in Eq. (8) was omitted in the implicit-moment simulation algorithm, which produced a small  $O(\beta\Delta t_i)$  inaccuracy in the simulation results because  $j_{e\parallel}^n$  was determined by the correct particle equations of motion.

We sincerely regret these errors and apologize for any confusion that this may have caused. This work was performed for the U.S. Department of Energy under Contract No. W-7405-ENG-48 at the University of California Lawrence Livermore National Laboratory and contributed to the Numerical Tokamak Turbulence Project, which was an activity supported jointly by the U.S. Department of Energy’s Office of Fusion Energy Sciences and the Mathematics, Information, and Computational Sciences Division as part of the High Performance Computing and Communications Program.